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DAMPING OF THE TORSION STEM OF A TUNING FORK GYROSCOPE [U]

by

G. H. Hunt, B.Sc., Ph.D.

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SUMMARY

The Coriolis Torques produced by the application of a rate of turn to a tuning-fork gyroscope are measured by the response of a tuned torsion stem. Methods of damping the motion of this stem and the optimum magnitude of this damping are examined theoretically.

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1 INTRODUCTION

The tuning-fork gyroscope is one example of a class of gyroscopes in which rotation of a vibrating element relative to inertial space induces oscillatory Coriolis forces. These forces have the frequency and phase of the velocity of the vibrating element, and one of the principal problems in the development of usable instruments of this type has been the measurement of these small forces.

All instruments which have so far been extensively studied have been of the tuning fork type; a description of the design and performance characteristics of one of these instruments has been given by Hobbs[1]. The Coriolis forces due to the two tines of the tuning fork have the same magnitude but opposite direction and therefore form a torque, and this torque is measured by mounting the base of the fork on a torsion stem such that the natural frequency of torsional oscillation of this system is exactly equal to the tuning fork frequency. The Coriolis torque then drives the torsional system at exactly its resonant frequency, and the amplitude of oscillation in the steady state is only limited by the damping of the system. The oscillations are then detected by suitable transducers, normally electromagnetic pickoffs, and the output signals from these are a measure of the rate of turn of the instrument. Fig.1 shows the instrument schematically.

The performance of this torque-detecting system is very largely a function of the damping of the torsional oscillations. Low damping will give large amplitude of oscillations in the steady state, and thus if the pickoffs are a limiting factor in the sensitivity such low damping is desirable. But the smaller the damping, then the longer is the time taken for the system to respond to transient changes in the torque level, and thus for a useful instrument the damping should be reasonably large. In addition, if the damping is too low, the steady state response of the torsion system is very critically dependent on frequency match between the tuning fork and the torsion system, and for a number of reasons this frequency match may be difficult to maintain.

The natural damping of a torsion system in an instrument of the type described by Hobbs[1] is defined by its "Q" which is typically in the range 5,000 to 10,000. It will be shown later that the time constant of response for a frequency of fork oscillation of 900 cycles per second is then 2 to 4 seconds, which is impossibly long for many applications of rate gyroscope. For this reason experiments have been made with much lower values of "Q" which have been produced by deliberately applying additional electrostatic damping forces. The experiments have been very successful, and the performance of forks provided with this damping will be detailed in a separate Report. This Note examines theoretically the various factors relevant to the choice of the optimum magnitude of damping.

2 RESPONSE OF A DAMPED TORSION STEM TO OSCILLATORY TORQUES

Fig.1 shows schematically the tuning-fork gyroscope which is used as a model for the theoretical examination of the response to rates of turn.

The moment of inertia of the tuning-fork about the torsion axis is assumed to vary sinusoidally at the resonant frequency of the fork $n/2\pi$. The angular momentum of the fork about that axis is then

$$(I_0 + I_1 \sin nt)\,(\dot{\theta} + \Omega)$$

where θ is the rotation of the fork relative to the base, i.e. the twist of the torsion stem. Ω is the angular velocity of the base relative to inertial space.

The torques which are exerted on the fork arise from the elastic stiffness of the torsion stem giving a torque $-k\theta$, and from various damping effects giving a torque $-D\dot{\theta}$. Equating these to the rate of change of angular momentum,

$$D\dot{\theta} + k\theta + \frac{d}{dt}\left[(I_o + I_1 \sin nt)\,(\dot{\theta} + \Omega)\right] = 0 \qquad (1)$$

from which

$$(I_o + I_1 \sin nt)\ddot{\theta} + (D + I_1 n \cos nt)\dot{\theta} + k\theta = -(I_o + I_1 \sin nt)\dot{\Omega} - I_1 n \cos nt\, \Omega \;. \qquad \ldots(2)$$

Provided the base of the fork is very heavy, the torques exerted on the base by twisting of the torsion stem will not produce significant angular motion. The rubber mount of the base relative to the case prevents the transmission of high frequency oscillatory motion from case to base. Thus for a range of frequencies from zero to about 20 c/s, Ω may be equated to motion of the case, and from 20 c/s upwards Ω is effectively zero.

The left-hand side of equation (2) may then be regarded as defining the characteristics of the resonant torsion-stem system which will respond to the torques provided by the right-hand side. As it is a sharply tuned resonant system, it will not respond to the torque $-I_o\dot{\Omega}$ which may now be neglected. Similarly the terms in I_1 on the left-hand side are in general negligibly small and may also be neglected. The equation then reduces to

$$I_o\ddot{\theta} + D\dot{\theta} + k\theta = -I_1 \sin nt\, \dot{\Omega} - I_1 n \cos nt\, \Omega \;. \qquad (3)$$

2.1 Steady state response

For a steady rate of turn Ω, equation (3) becomes

$$I_o\ddot{\theta} + D\dot{\theta} + k\theta = -I_1 n \cos nt\, \Omega \qquad (4)$$

and the steady state response is

$$\theta = -\frac{I_1 n \Omega \sin (nt + \alpha)}{\sqrt{(k - I_o n^2)^2 + D^2 n^2}} \qquad (5)$$

where

$$\alpha = \tan^{-1} \frac{k - I_o n^2}{Dn} \;.$$

A typical amplitude and phase plot near resonance is shown in Fig.2. A condition of resonance occurs for $k = I_o n^2$, the response then being

$$\theta = -\frac{I_1 \Omega}{D} \sin nt \quad , \quad \alpha = 0 \ . \tag{6}$$

The angular oscillation of the torsion stem is then in phase with the motion of the tuning fork.

The equations are more easily studied by introducing the natural frequency $\omega/2\pi$ of the torsion system, and defining its damping by its Q. Then

$$D = \frac{I_o \omega}{Q} \quad , \quad k = I_o \omega^2 \ .$$

The steady state response is then

$$\theta = -\frac{I_1}{I_o} \frac{n\Omega \sin (nt + \alpha)}{\sqrt{(\omega^2 - n^2)^2 + \omega^2 n^2 / Q^2}} \quad , \quad \alpha = \tan^{-1} \frac{\omega^2 - n^2}{\omega n} Q \tag{7}$$

and resonance occurs for $\omega = n$ giving

$$\theta = -\frac{I_1 Q}{I_o} \frac{\Omega}{n} \sin nt \ . \tag{8}$$

The values of Q for forks of the type shown in Fig.1 are typically 5,000 to 10,000. Damping is caused by a number of factors, principally the effects of induced magnetic fields in the torsion pickoffs, and the natural damping of the rubber mount between base and case reflected into the torsion system. For Q values in this range, operation is normally very near the peak of the resonance curve, and small relative variations of n and ω result principally in a phase shift of the response or change in the value of α, rather than in a significant amplitude change. If the torsion stem is adjusted near resonance so that $\omega = (n + \Delta n)$ where Δn is small, the corresponding phase error is

$$\alpha = 2Q \, \Delta n / n \quad \text{approximately.} \tag{9}$$

Now if the phase of the response is to be known to 1/100 radian or ½ degree, which is desirable in order that discrimination may be made against errors due to tuning fork asymmetries, then for Q = 10,000 the maximum permissible relative frequency change is

$$\frac{\Delta n}{n} = \frac{\alpha}{2Q} = 5 \times 10^{-7} \ . \tag{10}$$

There are two mechanisms which cause a relative change of natural frequency of torsion stem and fork. The first is the effect of gravity or acceleration; if this acts along the torsion stem axis it produces a frequency change of the fork which is typically of magnitude

$$\frac{\Delta n}{n} = 4 \times 10^{-6} \text{ per } g \ .$$

The second effect is due to temperature change. The relative change of frequency of fork and torsion systems with temperature for steel is approximately

$$\frac{\Delta n}{n} = 1\cdot 5 \times 10^{-5} \text{ per } {}^{\circ}\mathrm{C} .$$

Corresponding acceleration and temperature changes in order to keep to the condition of equation (10) would then be:-

Acceleration less than 0·12 g
Temperature " " 0·03 °C .

For both reasons it is thus desirable to decrease the value of Q by increasing damping, allowing a much larger frequency change and correspondingly larger changes in acceleration and temperature.

2.2 Transient response

Equation (3) may again be used as a sufficiently accurate approximation to the exact equation (2) provided the restriction is again made that input frequencies above about 20 c/s will not be transmitted through the rubber mount to the base. It will be assumed for the study of transient response that the torsion system is tuned to exact resonance with the tuning fork, i.e. that

$$n^2 = \omega^2 = k/I_o$$

and hence equation (3) may be re-written

$$\ddot{\theta} + \frac{n}{Q}\dot{\theta} + n^2\theta = -\frac{I_1}{I_o}(\dot{\Omega}\sin nt + \Omega n \cos nt) \quad . \qquad (11)$$

The only torques which are significant are those at the resonant frequency $(n/2\pi)$, and a solution of the form

$$\theta = A \sin nt + B \cos nt$$

will be substituted. Then

$$(\ddot{A}\sin nt + \ddot{B}\cos nt + 2\dot{A}n\cos nt - 2\dot{B}n\sin nt)$$
$$+\frac{n}{Q}(\dot{A}\sin nt + \dot{B}\cos nt + An\cos nt - Bn\sin nt)$$
$$= -\frac{I_1}{I_o}(\dot{\Omega}\sin nt + \Omega n\cos nt)$$

....(12)

Equating sin nt and cos nt terms separately,

$$\left(\ddot{A} - 2\dot{B}n + \frac{\dot{A}n}{Q} - \frac{Bn^2}{Q}\right) = -\frac{I_1}{I_o}\dot{\Omega} \tag{13}$$

$$\left(\ddot{B} + 2\dot{A}n + \frac{\dot{B}n}{Q} + \frac{An^2}{Q}\right) = -\frac{I_1}{I_o}n\Omega \; . \tag{14}$$

The steady-state response is thus $A = -\frac{I_1}{I_o} Q \frac{\Omega}{n}$ as derived in equation (8). It may also be noted that irrespective of transients, given zero initial conditions then the integrated value of the signal A over a period t is

$$\int_o^t A = -\frac{I_1}{I_o}\frac{Q}{n}\int_o^t \Omega + T \tag{15}$$

where T represents transients at the time t. Thus provided these transients are kept sensibly small, the integrated in-phase output is a measure of the angle through which the instrument has rotated.

The transient behaviour of the system may be examined by eliminating B from equations (13) and (14), giving

$$\left[p^3 + \frac{n}{Q}p^2 + 2n^2p + \frac{n^3}{Q} + \frac{n^2p^4}{p^3 + \frac{n}{Q}p^2 + 2n^2p + \frac{n^3}{Q}}\right]A = \frac{I_1}{I_o}(p^2+n^2)\Omega$$

....(16)

where the p notation is used to represent the differential operator. Then at frequencies much lower than n, such that $p \ll n$, the equation is effectively simplified to

$$\left[\frac{n}{Q}p^2 + 2n^2p + \frac{n^3}{Q}\right]A = \frac{I_1}{I_o}(p^2+n^2)\Omega$$

or

$$A = \frac{I_1}{I_o}\cdot\frac{Q}{n}\,\frac{(p^2+n^2)}{(p^2+2nQp+n^2)}\,\Omega \; . \tag{17}$$

The physical meaning of this simplification is that B is effectively zero and that all signals, transient and steady-state, are of the form A sin ωt. Equation (17) then gives the transient response of A to Ω. It is seen that for $p \ll n$, there is effectively a delay in the response of A due to a first-order time constant

$$\tau = \frac{2Q}{n} \; . \tag{18}$$

This is equal to the normal time-constant of the second order equation (11), which is the time for oscillations at the resonant frequency ($n/2\pi$) to decay to $1/e$ of their amplitude. For many applications a short response time is desirable. Using typical values for a practical instrument $Q = 10{,}000$ and $n = 2\pi \times 900$ radians per second,

$$\tau = 4 \text{ secs} .$$

Again a much smaller value of Q would appear desirable, and this must be provided by higher damping. Methods of providing this damping will be discussed in the next section.

3 POSSIBLE METHODS OF INCREASING DAMPING

The essential requirement of any method of increasing damping is that it shall provide a torque on the fork base relative to the base of the instrument which is accurately in phase with the velocity of twist of the torsion stem and proportional to that twist. Such a torque would then form a part of the $D\dot{\theta}$ term of equation (2). Possible methods of providing this damping torque fall naturally into three classes, namely,

(a) Those in which damping is provided by mechanical means such as viscous liquids.

(b) Those in which damping is provided by electric or magnetic fields, but without those fields being provided by separate measurement of the torsion stem twist. An example of this might be eddy-current damping.

(c) Those in which the torsion stem twist is measured and suitably amplified and then applied as a feedback torque of the correct phase and amplitude.

The third method has a number of advantages, of which the most important are:-

(a) Only torsional motion is damped. It may be arranged that lateral motion of the fork base is not detected by the torsion pickoff, and no torsional motion can then result from it. By contrast it is difficult with methods (a) and (b) to arrange that lateral motion cannot be transformed by the damping medium into torsional forces.

(b) If the electrical output is accurately proportional to the applied damping forces, which is not difficult to achieve, then if the feedback loop is tight, the output will also be proportional to the rate of turn and this proportionality will not depend critically on the torsion stem characteristics or on pickoff linearity.

For these reasons attention has been concentrated on the feedback method of damping which is further considered in section 4.

4 DAMPING OF TORSION SYSTEM BY FEEDBACK

The feedback loop is shown schematically in Fig.3. The velocity of the torsion stem oscillation is assumed to be accurately measured by the pickoff, but the pickoff is also assumed to produce noise with an amplitude equivalent to an angular velocity N. The pickoff output is then fed through an amplifier onto a torque-producing device such that the overall torque/angular velocity gain is -A.

$$\text{Thus feedback torque} = M_F = -A(\dot{\delta} + N) \quad . \tag{19}$$

The torsion stem characteristic is, from equation (3)

$$I_o\ddot{\theta} + \frac{I_o\omega}{Q}\dot{\theta} + I_o\omega^2\theta = M_S + M_F \tag{20}$$

where $M_S = -I_1(\sin nt\,\dot{\Omega} + n\cos nt\,\Omega)$, this being the signal torque applied by the fork to the torsion stem.

The output signal S is assumed to be the pickoff output $(N+\dot{\theta})$ which is accurately proportional to the feedback torque.

Eliminating M_F from (19) and (20),

$$\theta = \frac{M_S - AN}{I_o p^2 + \left(\frac{I_o\omega}{Q} + A\right)p + I_o\omega^2} \tag{21}$$

and the output signal S is thus

$$S = M_S \frac{p}{I_o p^2 + \left(\frac{I_o\omega}{Q} + A\right)p + I_o\omega^2} + N\frac{I_o p^2 + \frac{I_o\omega}{Q}p + I_o\omega^2}{I_o p^2 + \left(\frac{I_o\omega}{Q} + A\right)p + I_o\omega^2} \tag{22}$$

The feedback loop thus affects differently the response to torque and to pickoff noise. Considering first the response to signal torque M_S, then it is clear that the complete system now acts as a torsion stem with a higher damping characterised by a new value Q' such that

$$\frac{1}{Q'} = \frac{1}{Q} + \frac{A}{I_o\omega} . \tag{23}$$

The natural frequency of the torsion system remains unchanged. Corresponding to this decrease of Q' the transient response time discussed in section 2.2 is similarly reduced to

$$\tau' = \frac{2Q'}{n} \tag{24}$$

and the steady state sensitivity discussed in section 2.1 is now

$$\theta = -\frac{I_1}{I_o} Q' \frac{\Omega}{n} \sin nt . \tag{25}$$

Similarly the phase-shift α for a frequency mis-match is now

$$\alpha = 2Q' \frac{\Delta n}{n} . \tag{26}$$

The effect of the feedback loop on the noise may also be seen from equation (22). Since even with feedback the Q of the system is generally of the

order of 100, only a very small fraction of the noise will be affected by the coefficient of N. For noise more than a few cycles different from the resonant frequency, this coefficient will be equal to unity. A very small proportion of the noise will be reduced by the loop, but this proportion is hardly significant. Equation (22) is effectively therefore

$$S = M_S \frac{p}{I_o p^2 + \frac{I_o \omega}{Q'} p + I_o \omega^2} + N \tag{27}$$

which for the steady state becomes

$$S = -\frac{I_1}{I_o} Q' \,\Omega \cos nt + N\,. \tag{28}$$

Thus the signal/noise ratio for a given rate Ω, being the ratio of the first and second terms of equation (28), is proportional to Q'. This provides the limit to which Q' can be reduced and the corresponding effects of response time and phase-shift also reduced.

The effect can also be examined of lateral motion of the base of the fork at the frequency n being seen by the torsion pickoff as a torsional motion. This can arise due to asymmetrics in the component parts of the complete pickoff. Such a signal can be represented by a pickoff noise N_1 in addition to the noise N already examined. Again using (22), equation (27) is now modified to

$$S = M_S \frac{p}{I_o p^2 + \frac{I_o \omega}{Q'} p + I_o \omega^2} + N + N_1 \frac{\omega^2 - n^2 + \frac{j\omega n}{Q}}{\omega^2 - n^2 + j\frac{\omega n}{Q'}} \tag{29}$$

which in the steady state and with $n = \omega$ reduces to

$$S = -\frac{I_1}{I_o} Q'\Omega \cos nt + N + N_1 \frac{Q'}{Q}\,. \tag{30}$$

Thus the lateral motion seen by the complete loop is attenuated in the ratio Q'/Q, the same ratio as for the signal torque, and the total effect of this error is thus not affected by the application of the feedback loop.

5 PRACTICAL FACTORS AFFECTING THE CHOICE OF Q'

It is desirable that Q' shall be a minimum for a short response time and for small phase errors due to frequency mis-match, and shall be a maximum for high signal to noise ratio. The practical limits are calculated below for a tuning-fork gyro of the A5/KH6 type, with a natural frequency of about 900 cycles per second ($n = 5000$).

5.1 Time constant

The maximum time constant is assumed 1/10 second.

Then maximum $Q' = \frac{\tau n}{2} = 250$.

5.2 Phase error

Frequency change with acceleration is 4×10^{-6} per g; with temperature is $1 \cdot 5 \times 10^{-5}$ per °C. The maximum value of $\Delta n/n$ is assumed 2×10^{-5}, corresponding to 1°C or 5g. The maximum allowable phase error is assumed 1/100 radian.

Then maximum $Q' = \dfrac{a}{2\ \Delta n/n} = 250.$

5.3 Signal-to-noise ratio

The limiting factor in the noise is the level at which it saturates the phase sensitive rectifier; with the existing instrument saturation occurs for a noise level 10 times the signal level at full scale.

It is assumed that the full-scale deflection required on the most sensitive range is 10°/hour, so that about 1/10°/hour can be seen, and about 1/5°/hour read accurately. Thus the maximum noise level is 100°/hour equivalent.

The A5/KH6 forks with permanent magnet pickoffs have a sensitivity of 5×10^{-3} µV per °/hour per Q', and the noise level from the pickoffs is about 50 µV.

Knowing the noise level as 50 µV and the maximum noise level as 100°/hour equivalent, then minimum sensitivity = $\frac{1}{2}$ µV per °/hour. Thus minimum $Q' = \frac{1}{2}/5 \times 10^{-3} = 100$.

It is seen that the practical limits on the effective Q' are very close with the parameters quoted above, and that shorter time constants which may be desirable can only be achieved by reduced pickoff signal/noise ratio, by increasing the fork amplitude to give higher torque/rate of turn sensitivity, or by accepting a reduction in the minimum detectable rate-of-turn.

6 METHODS OF APPLYING THE FEEDBACK DAMPING TORQUES

The essential requirement of the feedback system is that the feedback torque shall be an accurately known and preferably linear function of a current or voltage in the system. The two methods by which a torque may be applied are electrostatic and magnetic; it is possible to produce an alternating electrostatic force proportional to an applied alternating voltage or an alternating magnetic force proportional to an applied alternating current. The choice between the two methods rests largely on the required magnitude of force or torque; for low torques the electrostatic method is ideal since the physical construction of the system is very simple and pickup and cross-torques can be made very small. However the torques which may thus be applied are in general smaller than can be reached by a magnetic system, which has disadvantages of heating effects due to current being passed through the coils, the possibility of higher damping and higher pickup to and from the driving system, and a more complicated and less stable structure.

The deciding factor in the choice between the two systems is thus the required amplitude of feedback torque. This may be calculated in simple terms by considering a very simple model of the fork. Suppose the tines are idealised as point masses m, moving about a radius r from the torsion axis with amplitude $\Delta r \sin nt$. Then

$$\text{velocity of each mass} = \Delta r\, n \cos nt$$

$$\text{acceleration " " "} = -\Delta r\, n^2 \sin nt$$

Let Q_F be the Q of the fork. Then driving force F_D on each mass is

$$F_D = -\frac{m \, \Delta r \, n^2}{Q_F} \sin nt . \qquad (31)$$

Let the applied rate of turn to the instrument be Ω. Then the Coriolis force on each mass is

$$F_C = 2m \, \Delta r \, n \cos nt . \qquad (32)$$

The ratio of the amplitudes of these two forces is

$$\frac{|F_C|}{|F_D|} = \frac{2Q_F\Omega}{n} . \qquad (33)$$

For the A5/KH6 fork already considered, n = 5000 rad/sec and a typical Q_F value is 10,000. Thus

$$\frac{|F_C|}{|F_D|} = 4\Omega$$

where Ω is the rate of turn in radians per second. The Coriolis force is equal to the feedback force, given that this is applied at a radius equal to the radius r of the masses and that the feedback damping is much higher than the natural damping of the system ($Q >> Q'$).

Due to the larger mass and areas available for the application of feedback force compared with driving force, then for a similar mechanism of providing the two forces, and with similar currents or voltages applied to each, it should be possible to make $|F_C|/|F_D|$ equal to 4. Thus a maximum rate of turn of 1 radian per second or about 2×10^5 degrees per hour could be measured before the feedback system saturated.

Thus provided a rate of about 1 radian per second can be accepted as a maximum, it would appear that an electrostatic feedback system can be designed for an electrostatically driven fork. For the larger amplitudes of fork motion possible with a magnetically driven fork, magnetic feedback will be necessary.

These conclusions have been confirmed by preliminary measurements with an electrostatic system and rates of turn up to $\frac{1}{2}$ radian per second have been applied without causing saturation or detectable non-linearity.

A factor which could lead to errors in a feedback loop designed to operate at high rates of turn is possible variation of the feedback forces and thus of the scale factor due to variation in the geometry of the electrostatic or magnetic force systems. This could be caused by bending of the structure with applied acceleration, and may limit the size of gap between feedback plates or in the magnetic circuit. This in turn could provide a limit on the maximum feedback torque and thus on the maximum rate of turn.

7 CONCLUSIONS

The necessity for damping the torsion stem of a tuning fork gyro arises from requirements of short response-time and a frequency match between

torsion stem and tuning fork which is not impossibly critical. Calculations show that a torsion system Q in the range 100-250 is desirable. It is shown that the feedback method of providing this damping offers advantages of linearity and will not transform lateral unbalance motion of the fork into damping torques which would be interpreted as a rate of turn.

It is shown that electrostatic feedback can be used at rates of up to 1 radian per second with an electrostatically driven fork, but that with the higher amplitudes of magnetically driven forks a magnetic feedback may be necessary.

The principal disadvantage of any method of increasing the damping is the reduced angular motion of the torsion stem for a given rate of turn; this in practice sets the lower limit on the range of Q's which may be used.

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ATTACHED:-

Appendices 1 and 2
Drgs. IEE.124-126
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APPENDIX 1

EFFECT OF ACCELERATION ON TUNING-FORK FREQUENCY

Consider a tuning fork whose tines have effective length L and effective mass m. Let the tine be displaced through an angle θ, and consider the restoring forces on the mass m which has been displaced through $L\theta$.

$$\text{Inertial force} = m\frac{d^2}{dt^2}(L\theta) = mL\ddot{\theta}$$

$$\text{Elastic restoring force} = k\theta$$

$$\text{Acceleration force} = ma\theta$$

where a is the acceleration parallel to the undisplaced tine axes.

The equation of motion of the tine is therefore

$$mL\ddot{\theta} + (k+ma)\theta = 0 .$$

The natural frequency of oscillation is given by

$$n^2 = \frac{k+ma}{mL}$$

and hence if the natural frequency under zero acceleration is given by n_o,

$$n^2 - n_o^2 = \frac{a}{L}$$

from which the proportional frequency change is

$$\frac{\Delta n}{n} = \frac{1}{2n^2}\frac{a}{L} \quad \text{approx.}$$

For the A5/KH6 size fork with L = 4 cm, n = 5000 approx., the above equation gives,

$$\frac{\Delta n}{n} = 5 \times 10^{-6} \text{ per g} .$$

This value has been confirmed experimentally.

A more detailed treatment by Karolus and Thiesbürger[2] shows that the effective length L of the tine is about 0·7 of the total length, depending on the actual geometrical shape of the tine.

APPENDIX 2

EFFECT OF TEMPERATURE ON TUNING-FORK AND TORSION FREQUENCIES

The natural frequency of a tuning fork made from material of density ρ and Youngs Modulus E is given by

$$n = \frac{b}{L}\sqrt{\frac{E}{\rho}}$$

where b is a number determined by the geometrical shape of the tines and L is the effective length. For a given fork, temperature variations will change the Youngs Modulus, length and density, but will not change the mass, and it is therefore convenient to use the alternative form

$$n = c\sqrt{\frac{EL}{m}}$$

where m is the tine mass, c a numerical factor related to b. The effect of temperature variations is then

$$\frac{1}{n}\frac{\partial n}{\partial T} = \frac{1}{2}\left(\frac{1}{E}\frac{\partial E}{\partial T} + \frac{1}{L}\frac{\partial L}{\partial T}\right).$$

Similarly for a torsion oscillator, the frequency may be expressed by

$$\omega = d\sqrt{\frac{GL}{m}}$$

where G is shear modulus, L the length of torsion stem, and d a geometrical factor dependent on the dimensions of torsion stem and of the inertial masses. Thus

$$\frac{1}{\omega}\frac{\partial \omega}{\partial T} = \frac{1}{2}\left(\frac{1}{G}\frac{\partial G}{\partial T} + \frac{1}{L}\frac{\partial L}{\partial T}\right).$$

Figures given by Kaye and Labye for steel are

$$-\frac{1}{E}\frac{\partial E}{\partial T} = 2{\cdot}3 \times 10^{-4}\ {}^{\circ}C^{-1}$$

$$-\frac{1}{G}\frac{\partial G}{\partial T} = 2{\cdot}6 \times 10^{-4}\ {}^{\circ}C^{-1}$$

$$-\frac{1}{L}\frac{\partial L}{\partial T} = 10^{-5}\ {}^{\circ}C^{-1}$$

from which it is seen that the effects of moduli variations are much greater than those of length changes.

The corresponding theoretical frequency changes are:-

For a fork, $$-\frac{1}{n}\frac{\partial n}{\partial T} = 1\cdot 2 \times 10^{-4}\ {}^{\circ}C^{-1}$$

For a torsion system, $$-\frac{1}{\omega}\frac{\partial \omega}{\partial T} = 1\cdot 35 \times 10^{-4}\ {}^{\circ}C^{-1}$$

Difference of coefficients $= 0\cdot 15 \times 10^{-4}\ {}^{\circ}C^{-1}$

These values have been verified experimentally.

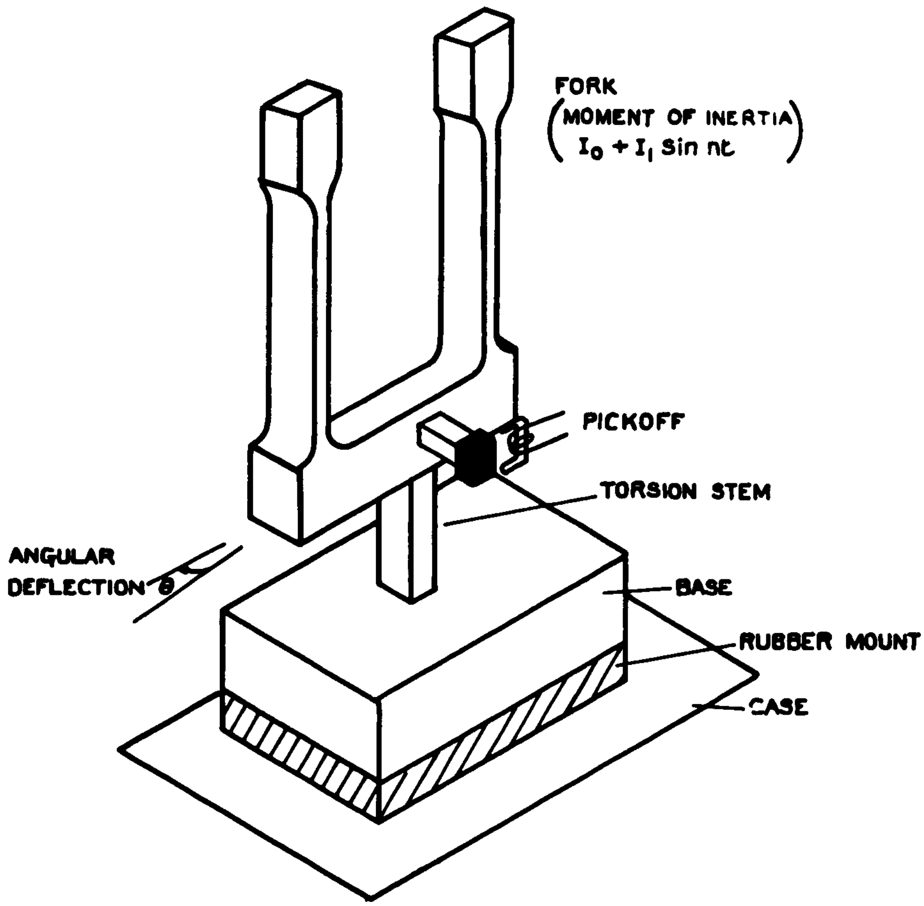

FIG. 1. SCHEMATIC INSTRUMENT.

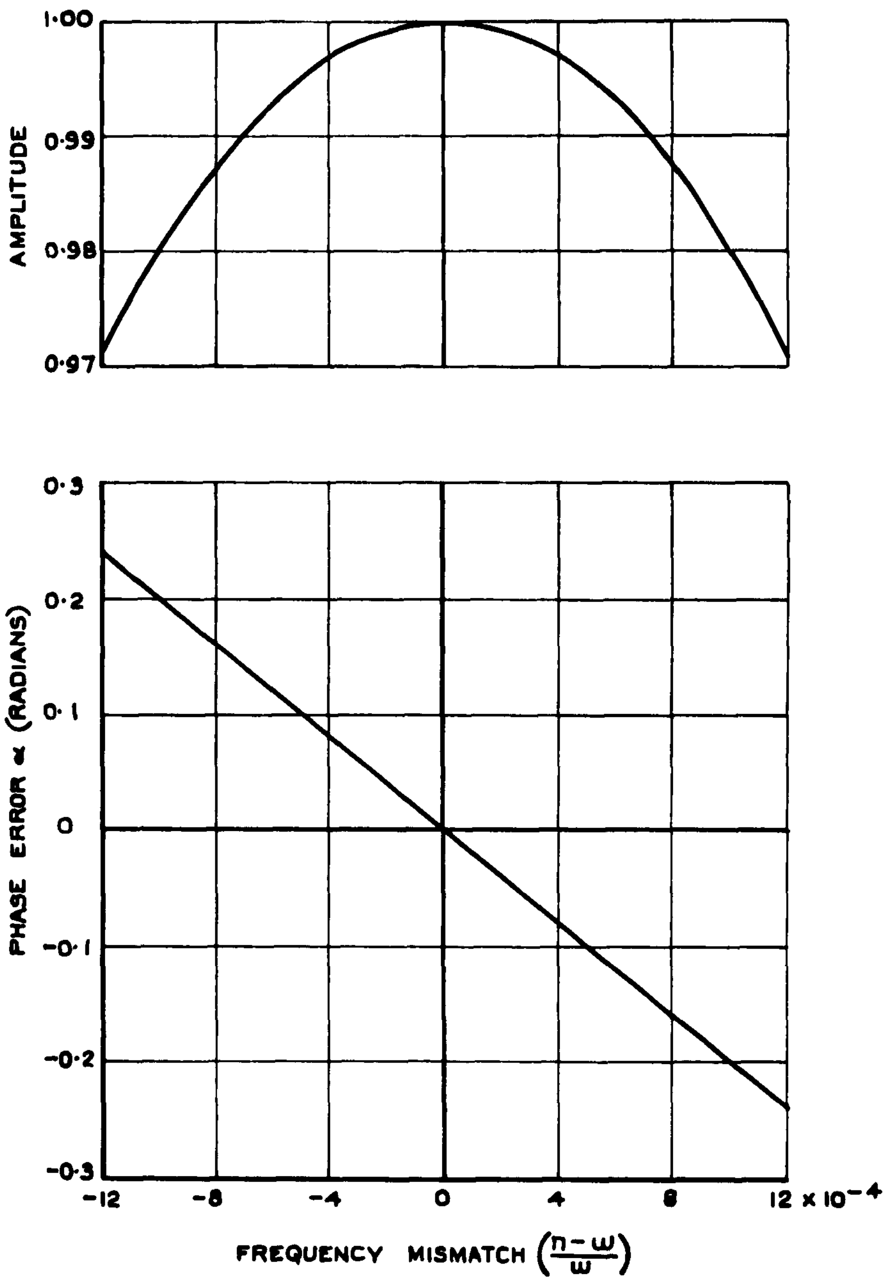

FIG. 2. AMPLITUDE AND PHASE RESPONSE NEAR RESONANCE (Q = 100)

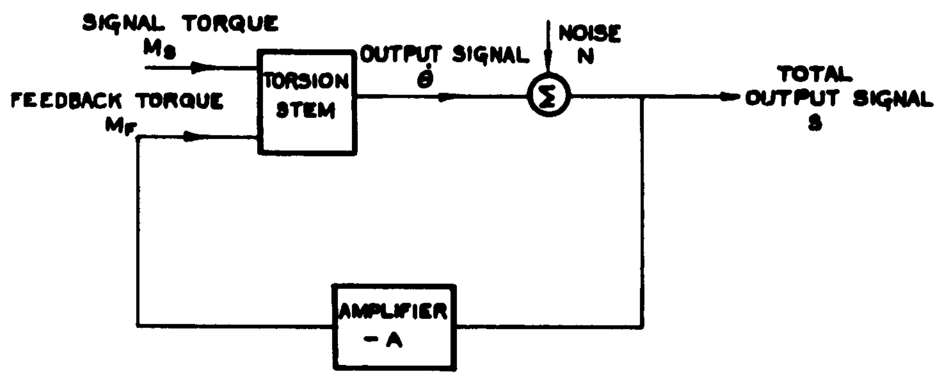

FIG. 3. TORSION FEEDBACK LOOP.

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